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Triaxiality, chirality, and γ-softness

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Abstract. Current work explores the impact of γ-softness on partner bands built on the $\pi h_{11/2} \nu h_{11/2}$ particle-hole configurations in triaxial odd-odd nuclei. The results of calculations conducted using a core-particle-hole coupling are presented. The model Hamiltonian includes the collective core, the single-particle valence nucleons, and separable quadrupole-quadrupole interactions. The Kerman-Klein method was applied to find eigenstates, which provided a convenient way for exploring core effects. Calculations were made for triaxial cores with various γ-softness using the General Collective Model keeping the expectation value for the triaxiality parameter fixed at $\langle \gamma \rangle = 30°$. The degeneracy in the $\pi h_{11/2} \nu h_{11/2}$ bands results from the calculations for the γ-rigid core but is lifted for the γ-unstable core.

While the impact of quadrupole deformation on the structure of nuclei away from the closed shell is well recognized [1], the existence of static triaxial shapes continues to be a subject of a long standing debate. The rigid triaxial- [2] and γ-unstable [3] rotor models yield very similar predictions for the yrast states in nuclei with even number of protons and neutrons (even nuclei) as illustrated by the calculations presented in Figs. 1 and 2.
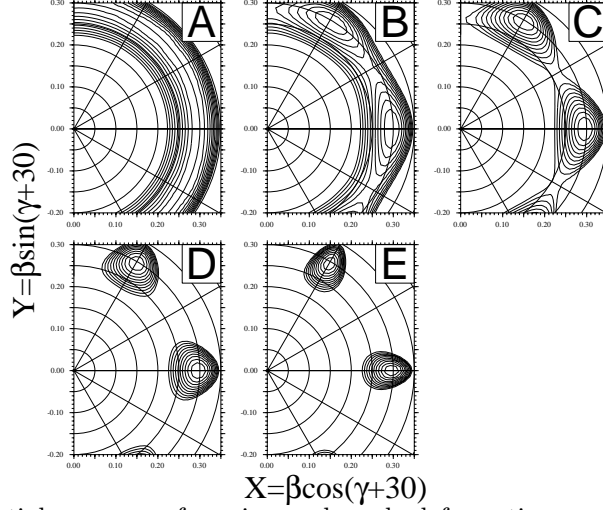
Fig. 1. Potential energy surfaces in quadrupole deformation coordinates β and γ for triaxial cores of varying γ-softness ($\langle\gamma^2\rangle$-$\langle\gamma\rangle^2$) with the expectation values of other deformation parameters such as $\langle\beta\rangle$, $\langle\beta^2\rangle$ and $\langle\gamma\rangle$ constrained to be nearly identical.

Figure 1 shows the potential energy surfaces in quadrupole deformation coordinates β and γ for triaxial cores of varying γ-softness ($\langle\gamma^2\rangle$- $\langle\gamma\rangle^2$) with the core labeled as **A** being γ-unstable, and cores labeled **B**-**E** becoming more γ-rigid; the potential energy surface for the rigid triaxial core labeled below as **F** is not shown in Fig. 1 since it corresponds to the Dirac's δ function at $\gamma = 30°$, $90°$, and $120°$. The above potential energy surfaces were constructed to differ by the γ-softness parameter only, meaning that the care was taken to assure that the expectation values of other deformation parameters such as $\langle\beta\rangle$, $\langle\beta^2\rangle$ and $\langle\gamma\rangle$ are nearly identical.

These potential energy surfaces were used to solve the collective Bohr Hamiltonian following the General Collective Model of Ref. [4] with the computer code described in Ref. [5]. The resulting energy levels are compared in 2. It can be observed in this figure that the yrast states, shown for each core in the corresponding column on the left hand side, are not effected by the γ-softness. Possible signatures of γ-softness or γ-rigidity from these calculations involve non-yrast states and are likely to be perturbed by single particle degrees of freedom which are not taken into account in the model Hamiltonian. As a consequence, conclusive results on stability of triaxial deformation in even nuclei are hard to obtain and seems beyond the sensitivity of the current experimental investigations.

The situation is not very different if the calculated yrast bands in odd-mass nuclei (odd nuclei) are examined. Odd nuclei can be considered as consisting of an even core with a coupled valence quasiparticle. The results presented in Fig. 3 for a core-quasiparticle model calculations following the Kerman-Klein method [6, 7] demonstrate essentially insignificant dependence of the yrast state energies in odd
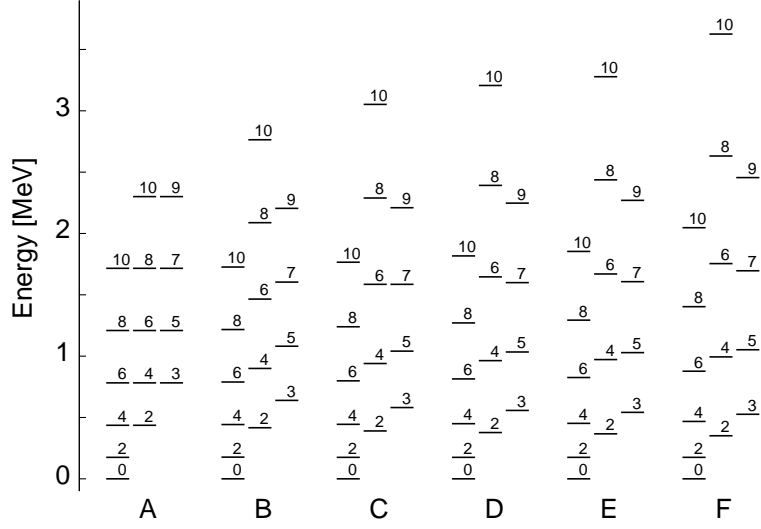
Fig. 2. Energy levels which result from the diagonalization of the Bohr Hamiltonian following the General Collective Model of Ref. [4] for (**A**-**E**) potential energy surfaces shown in Fig. 1 and (**F**) for the rigid triaxial rotor of Ref. [2].

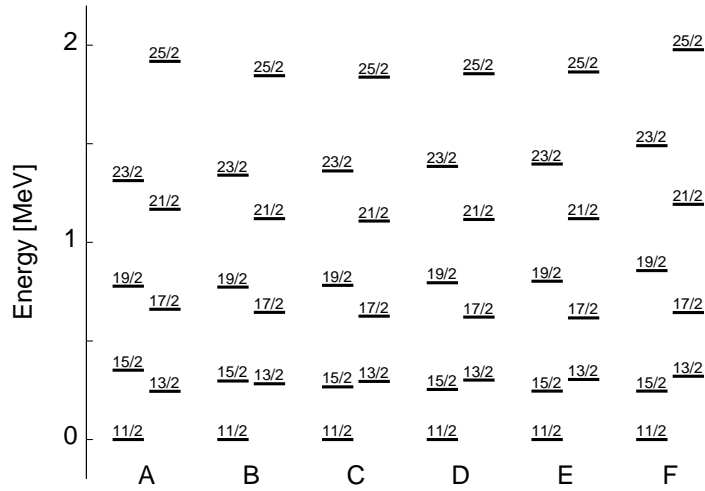**Fig. 3.** Energy levels in yrast bands of odd nuclei as resulting from a valence $h_{11/2}$ odd-particle or odd-hole coupling to the cores **A**-**F**. The calculations follow the Kerman-Klein method discussed in Ref. [6, 7].
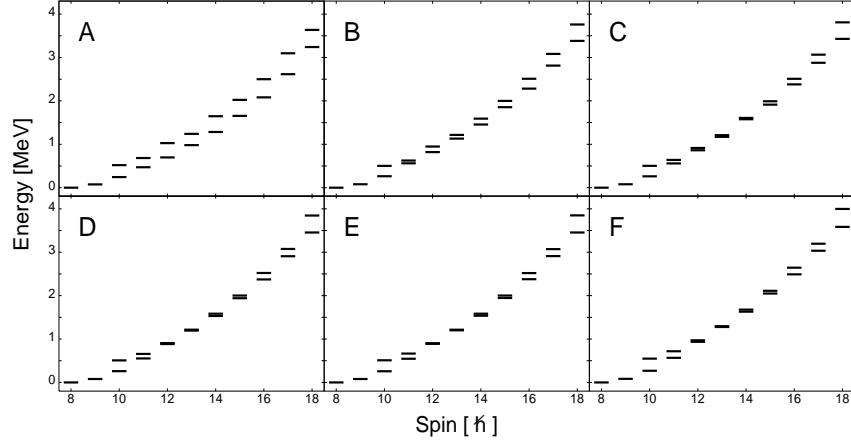
Fig. 4. Energy levels in yrast and yrare bands of doubly-odd nuclei as resulting from a valence odd-particle and odd-hole coupling in the $\pi h_{11/2} \nu h_{11/2}$ configuration to the cores **A**-**F**. The calculations follow the Kerman-Klein method discussed in Ref. [6, 7].

nuclei on the γ-softness of the **A**-**F** cores used for the coupling with an unique-parity $h_{11/2}$ state. The results shown in Fig. 3 are valid for a pure $h_{11/2}$ particle or hole coupling due to the particle-hole symmetry discussed in Ref. [8].

As a surprise, therefore, may come the observation that particle-hole coupling calculations which follow the model developed in Ref. [6, 7] for nuclei with odd number of protons and neutrons (doubly-odd nuclei) show sensitivity of the yrast and near yrast energy levels to the degree of γ-softness. This is illustrated in Fig. 4 presenting the two lowest energy states at a given spin calculated for the $\pi h_{11/2} \nu h_{11/2}$ configuration and the cores defined above. The separation between the states of the same spin in the mid-spin range near $\sim 14\hbar$ is observed to decrease with decreasing γ-softness by nearly an order of magnitude, from ~ 400 keV for the γ-unstable core **A** to ~ 40 keV for the γ-rigid core **F**. The proposed interpretation of this effect invloves a doubling of states resulting from chiral coupling of angular momenta vectors for stable triaxial core [6] and lack of such doubling for the γ-soft core for which the stable chiral geometry cannot be formed.

It should be pointed out here that the calculations presented in Fig. 4 are generic and should be treated as qualitative rather than quantitative. This results from the fact that the parameters of the model were not optimized to fit any particular nucleus. In addition the core basis had to be truncated at spin 10^+ as shown in Fig. 2; this truncation is imposed by the General Collective Model cores available to the authors. However, while the numerical values of the energy separation between levels of the same spin in the mid-spin range for doubly-odd nuclei may change depending on the details of the calculations the correlation of the γ-softness and the energy separation is expected to stay. The study of particle-hole configurations
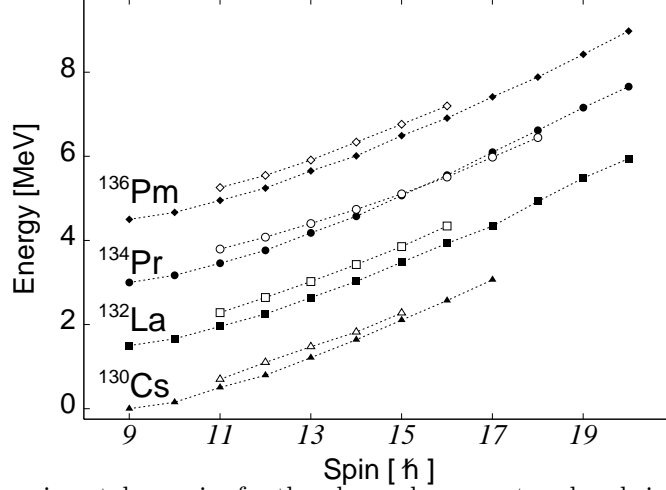
Fig. 5. Experimental energies for the $\pi h_{11/2} \nu h_{11/2}$ partner bands in N=75 doubly-odd isotones in the A$\sim$130 region. Bandhead energies are separated by 1.5 MeV for display.

in doubly-odd transitional nuclei, therefore, can shed light on the γ-stability of their triaxial cores.

As mentioned above, the small energy spacing between states of the same spin in doublet bands built on the particle-hole configuration has been proposed as a signature of formation of chiral geometry in triaxial doubly-odd nuclei [9]. Nuclear chirality is a manifestation of spontaneous symmetry breaking [10] resulting from an orthogonal coupling of angular momentum vectors in triaxial nuclei which minimizes the total energy of the system. For doubly-odd nuclei three perpendicular angular momenta provided by the valence particle, valence hole and the collective core rotation can form two geometries of the opposite handedness; these two geometries are related by the time reversal operator, which reverses an orientation of each of the angular momentum components.

Up to date a number of cases has been identified experimentally in the A$\sim$ 130 and the A$\sim$ 104 regions with the doublet bands built on the $\pi h_{11/2} \nu h_{11/2}^{-1}$ and the $\pi g_{9/2}^{-1} \nu h_{11/2}$ configurations, respectively. Figure 5 shows the Energy vs. Spin plot for four N=75 isotones discussed in Ref. [11]. The experimental data for the band's energetics seem to be in good qualitative agreement with the calculations shown in Fig. 4. For the quantitative agreement, however, the calculations need to be performed with more complete core basis and optimized set of model parameters.

One of the recent important observations related to the properties of the doublet bands in the mass 130 region is a notable difference in the electromagnetic transition rates between the doublet bands [9]. It seems very interesting to examine the γ-softness effects on the band's electromagnetic properties, however, the core basis truncation in the current studies limits the meaningfull comparison due to the fact

that the electromagnetic transition rates depend on the details of the wave function in more sensitive way than the state energies. Again, improved calculations with extended core space are needed.

In conclusion, the presented core-particle-hole coupling calculations indicate a correlation between the energy separation in doublet bands built on the unique-parity intruder states and γ-rigidity of the collective core in doubly-odd nuclei. Small energy separation between states of the same spin in the medium-spin range results from the calculations with a relatively γ-rigid core but this degeneracy is lifted if the core is γ-unstable. Experimentally there are two doubly-odd nuclei identified so far with small separation between doublet bands: ^{104}Rh in the mass ~ 104 region with the two 17^- states built on the $\pi g_{9/2}^{-1}\nu h_{11/2}$ configuration being 2 keV apart [12] and ^{134}Pr with the states at 15^+ and 16^+ built on the $\pi h_{11/2}\nu h_{11/2}^{-1}$ configuration being less than $\sim$50 keV apart [11] . Further, improved calculations are needed to examine a possible stable triaxiality of these nuclei. These improved calculations should also be examined for consistency of electromagnetic rates with the experimental data.

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